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Heat transfer in square cavities with partially active vertical walls

ALVARO VALENCIA and RAMÓN L. FREDERICK
Departamento de Ingeniería Mecánica, Universidad de Chile, Casilla 2777,
Santiago, Chile

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Abstract—Natural convection of air in square cavities with half-active and half-insulated vertical walls is numerically investigated for Rayleigh numbers of 10^3–10^7. This problem is related to applications in solar collection and cooling of electronic components. Five different relative positions of the active zones are considered. While circulation depends strongly on the total exit length downstream of the active zones, heat transfer depends less on this parameter. Significant conduction effects occur even at a Rayleigh number of 10^5. Expressions for average Nusselt number in the five situations and hints on how to generalize heat transfer results to intermediate cases are given.

1. INTRODUCTION

THE STUDY of laminar natural convection in rectangular cavities with two walls at imposed temperatures provides a useful description of the behaviour of confined fluids in many practical situations. However, departures from this basic situation are often encountered. In fields like solar energy collection and cooling of electronic components, the active walls may be subject to abrupt temperature nonuniformities due to shading or other effects. The relative position of the hot and cold wall regions has significant effects on the flow patterns and heat transfer. This paper describes the natural convection of air inside a square cavity with two vertical walls that are half-active (i.e. with imposed temperature over half of the wall length) and half-insulated.

Previous works on internal natural convection with mixed temperature conditions on the walls are relatively scarce. Churchill *et al.* [1] studied the effect of a heated strip located on the vertical wall of a rectangular enclosure. Natural convection in an inclined rectangular box with the lower surface half-heated and half-insulated has also been investigated [2] up to a Rayleigh number of 10^4. Average Nusselt numbers based on the heated area exceed those obtained for surfaces with a uniform wall temperature. The effect of relative positions of warm and cold sectors on the same vertical wall of a rectangular cavity has also been studied [3].

Kuhn and Oosthuizen [4] studied the transient heat transfer in an enclosure with a partially heated vertical wall and with the other vertical wall fully cooled. By moving the heater from top to bottom, *Nu* increases to a maximum and then decreases. Natural convection in a cavity with partial heating from below and with a vertical side cooled has been investigated numerically [5]. In these two works the importance of carefully determining Nusselt numbers, which are affected by heat flux singularities due to the mixed temperature conditions at an active wall, is stressed.

Five forms of the proposed problem are studied here numerically. As in other recent studies dealing with adiabatic–isothermal boundary conditions on a wall [4, 5], and in order to allow direct comparison with the case of a square cavity with active walls at uniform temperatures (test problem), two-dimensional flow is assumed. Attention is focused on the flow and heat transfer effects of the position of the active regions at Rayleigh numbers of 10^3–10^7. Expressions for average Nusselt numbers are proposed.

2. FORMULATION

The physical situation is shown in Fig. 1. A cavity of square cross-section is filled with air ($Pr = 0.71$). Half the height of the right-hand wall is kept at a temperature T_h. Also, half the height of the left-hand wall is at temperature T_c, with $T_h > T_c$. The remaining parts of the perimeter are insulated. The temperature difference is kept low, so that the Boussinesq approximation can be employed. Five different cases (problems) will be studied here. In case 1 (Fig. 1(a)), the hot region is located at the bottom and the cold region at the top of their respective walls. In case 2 (Fig. 1(b)), hot and cold regions are at the top right and bottom left regions, respectively. In cases 3 and 4 (Figs. 1(c) and (d)) both active sections are directly opposite to each other, at the lower half of the enclosure and at an intermediate position, respectively. Finally, in case 5 (Fig. 1(e)) the hot region is located at the right-hand wall from $Y = 0.25$ to 0.75, whereas the cold region occupies the bottom half of the left-hand wall.

The dimensionless governing equations in primitive form for two-dimensional, laminar, permanent flow

NOMENCLATURE

- b exponent in equation (6)
- C coefficient in equation (6)
- d total dimensionless exit length
- g gravitational acceleration [m s^{-2}]
- L cavity side [m]
- n inward normal to the surface
- $\overline{Nu}$ Nusselt number averaged on a vertical plane
- Nu_0 average Nusselt number referred to active area
- P dynamic pressure [N m^{-2}]
- Pr Prandtl number, ν/α
- Ra Rayleigh number, $g\beta(T_h - T_c)L^3/\nu\alpha$
- T temperature [K]
- T_h, T_c hot and cold wall temperatures [K]
- U, V dimensionless velocities in the X- and Y-directions
- X, Y dimensionless coordinates.

Greek symbols

- α thermal diffusivity of air [m^2 s^{-1}]
- β thermal expansion coefficient of air [K^{-1}]
- Θ dimensionless temperature, $(T - T_c)/(T_h - T_c)$
- ν kinematic viscosity of air [m^2 s^{-1}]
- ρ density of air [kg m^{-3}]
- ψ dimensionless stream function
- ψ_{max} maximum value of stream function.

Reference quantities

- L for coordinates
- ν/L for velocities
- ν for stream function
- $\rho(\nu/L)^2$ for pressure.

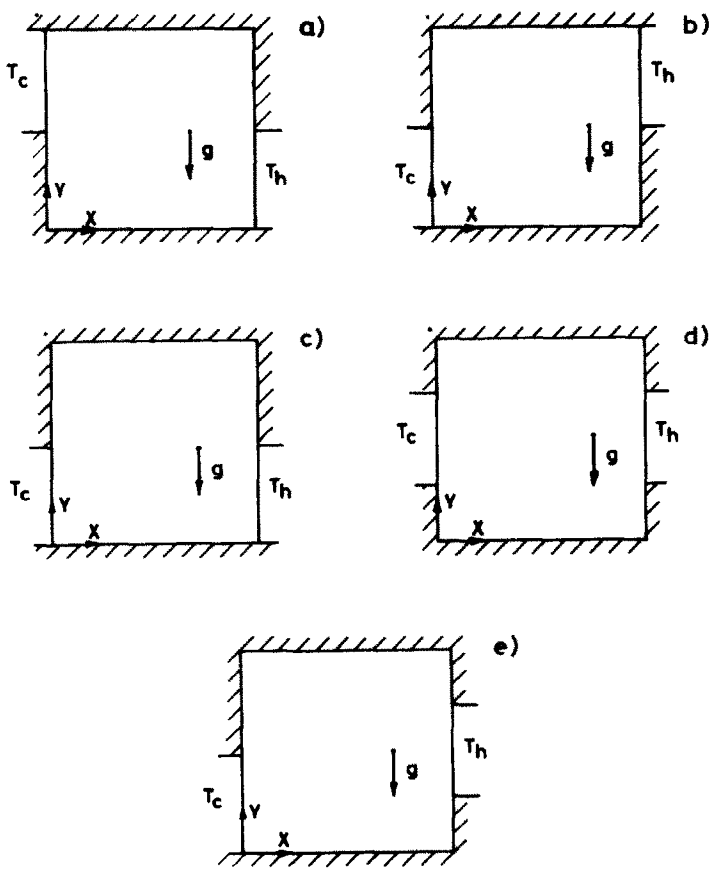

FIG. 1. Physical model.

of an incompressible Boussinesq fluid are:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$\frac{\partial (U^2)}{\partial X}+\frac{\partial (UV)}{\partial Y}=-\frac{\partial P}{\partial X}+\nabla^2 U \quad (2)$$

$$\frac{\partial (UV)}{\partial X}+\frac{\partial (V^2)}{\partial Y}=-\frac{\partial P}{\partial Y}+\nabla^2 V+(Ra/Pr)\Theta \quad (3)$$

$$\frac{\partial (U\Theta)}{\partial X}+\frac{\partial (V\Theta)}{\partial Y}=(1/Pr)\nabla^2\Theta. \quad (4)$$

On all walls $U = V = 0$. The temperature conditions are $\Theta = 1$ and 0 on the hot and cold parts of the active walls, respectively, whereas on the insulated walls $\partial\Theta/\partial n = 0$. Rayleigh numbers of 10^3–10^7 are studied. The flow is considered laminar over this range of Rayleigh numbers.

3. NUMERICAL METHOD

Equations (1)–(4) are solved by the SIMPLEC method [6]. This method, based on the control volume formulation, is appropriate for working with mixed boundary conditions. Uniform grids of 41×41 points were used throughout. To validate the numerical procedure, results for the test problem were compared with earlier results [7–9]. Our average wall Nusselt numbers differ from the benchmark values of ref. [7] by 6.5% at $Ra = 10^6$ and by 2.7% at $Ra = 10^5$. Our Nusselt numbers at $Ra = 10^7$ exceed those in ref. [8] by 2.7% and in ref. [9] by 12.2%. In ref. [9] a finite element method was used, and Nusselt numbers were extrapolated to zero element size.

Great care was taken in the determination of Nusselt numbers, as the mixed temperature conditions lead to heat flux singularities on both vertical walls. Local Nusselt numbers were first determined at the active parts of the walls and then averaged using Simpson's rule. In some cases, unacceptable differences were found between the averaged values at the hot and cold walls. To evaluate the effect of the singularity on the numerical solution, average Nusselt numbers at two vertical planes ($X = 0.25$ and 0.75) were calculated by the equation

$$\overline{Nu}=\int_0^1\left(Pr\,U\Theta-\frac{\partial\Theta}{\partial X}\right)\mathrm{d}Y. \quad (5)$$

The $\overline{Nu}$ values at these two positions differ by 2.5% or less in all computer runs reported here. When this criterion was applied to the test problem results, differences of the same order were found. After we made sure that the temperature field was not significantly distorted by the singularities, we evaluated overall heat transfer at the vertical midplane using equation (5). By referring overall heat transfer to the appropriate area in the five new cases, average Nusselt numbers (Nu_0) for the active wall regions were determined. Values of Nu_0 are believed to be free from the effect of wall singularities.

4. RESULTS AND DISCUSSION

The fives cases will be described in detail, then a generalization of the results will be given. A summary of results is shown in Table 1.

4.1. *Flow regimes*

In all cases at low Rayleigh number, the fluid rotates around the cavity midpoint. Isograms of the stream function at $Ra = 10^6$ (Figs. 2(a)–(e)), which are representative of the flow in the boundary layer regime, will be described.

In case 1, two clearly defined centres of rotation, adjacent to the insulated parts of the vertical walls, appear (Fig. 2(a)). The flow leaving the active region is free to move upwards. In case 2, boundary layers start at $Y = 0.5$. The flow is again divided into two cells, but the centres of rotation are now located near the active parts of the vertical walls (Fig. 2(b)). The flow out of the active regions is subject to severe restrictions imposed by the horizontal walls. Because of this, most of the fluid recirculates within the upper or lower halves of the enclosure, with only a weak

Table 1. Selected numerical results

Variable	Ra 10^3	10^4	10^5	10^6	10^7
Nu_0(T)†	1.123	2.291	4.699	9.580	19.497
Nu_0(1)	1.463	2.997	5.713	11.659	23.434
Nu_0(2)	1.428	2.142	3.295	4.505	6.132
Nu_0(3)	1.508	2.676	5.305	10.962	22.514
Nu_0(4)	1.795	3.399	6.383	12.028	22.973
Nu_0(5)	1.625	2.730	4.892	8.567	15.255
ψ_{max}(T)	1.671	7.088	13.390	22.730	38.230
ψ_{max}(1)	1.347	7.151	15.450	28.390	49.390
ψ_{max}(2)	1.021	3.332	4.723	7.936	9.788
ψ_{max}(3)	1.210	5.245	10.530	15.460	25.290
ψ_{max}(4)	1.442	5.923	10.080	15.640	24.130
ψ_{max}(5)	1.238	4.611	7.307	10.450	13.990

† Numbers in parentheses refer to case number (T = test problem).

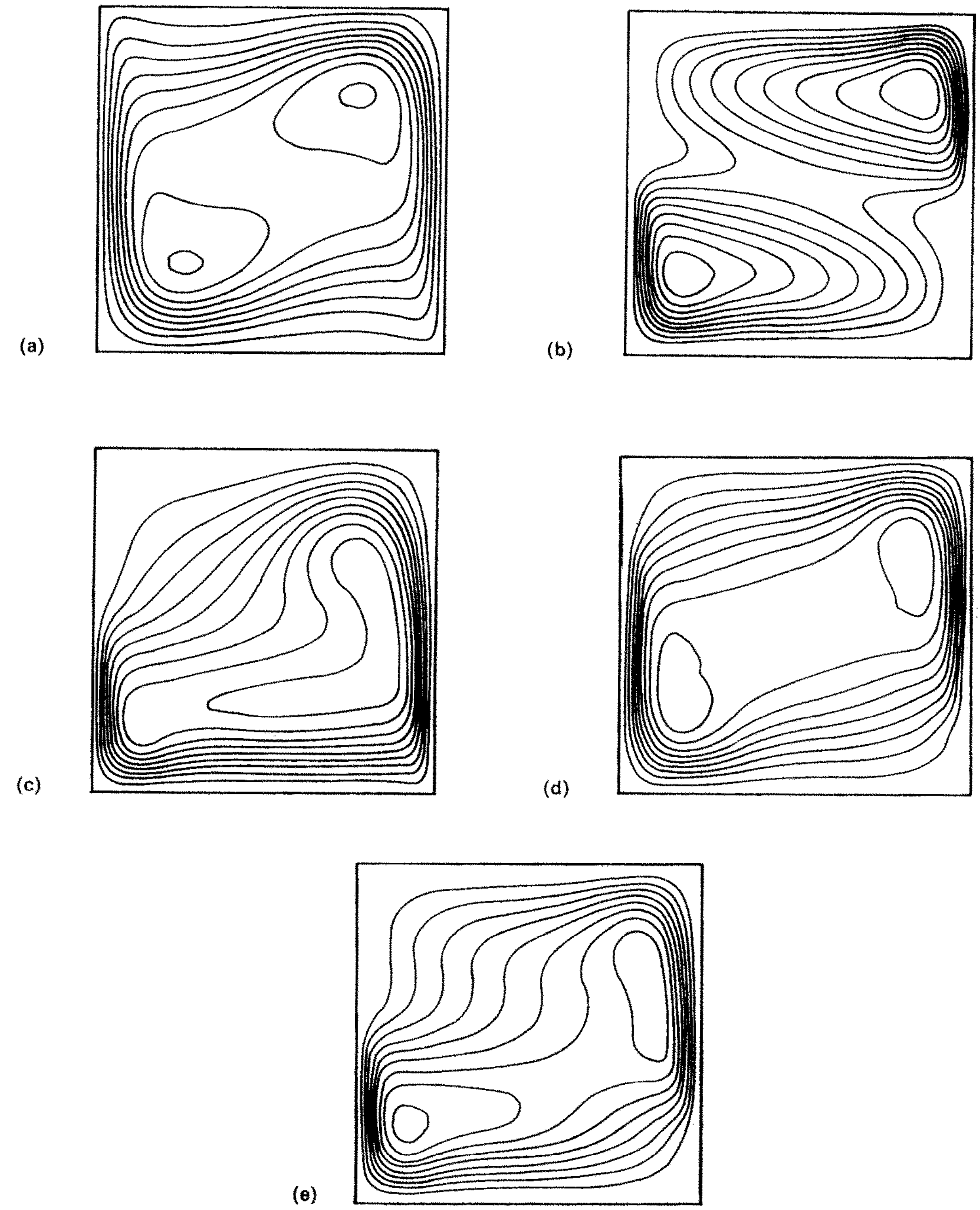

FIG. 2. Stream function contours, $Ra = 10^6$: (a) case 1, $\psi = 1.0$–28.0; (b) case 2, $\psi = 1.0$–7.2; (c) case 3, $\psi = 1.0$–14.2; (d) case 4, $\psi = 1.0$–14.5; (e) case 5, $\psi = 1.0$–10.0. Equally spaced streamlines.

circulation between the two halves. This pattern results in a much weaker flow than in the preceding case.

In case 3 (Fig. 2(c)), boundary layers form at the lower half of the cavity. The flow in the upper half does not have a boundary layer character. One of the boundary layers starts at a corner while the other starts at $Y = 0.5$. The fluid in the boundary layer on the hot side is free to move upwards from the end of the hot section, while the stream parallel to the opposite wall faces the restriction of a horizontal wall. Although the area that generates buoyancy is the same as in problem 1, one severe restriction to flow is added. In case 4 (Fig. 2(d)), the flow leaving an active wall region is free to move either upwards or downwards. Fluid circulation therefore fills the entire cavity even at very high *Ra*. In problem 5 (Fig. 2(e)), the cold side rotation is similar to that in problem 2, while in the hot side, the similarity to problem 3 or 4 is closer.

Circulation rates result from the balance between buoyancy forces and flow restrictions. The active area is the same in all problems, but the total dimensionless exit length downstream of the active zones (d) varies. In case 1 d has a value of 1; in problems 3 and 4 d has a value of 0.5; in problem 5 d has a value of 0.25; and in problem 2 d has a value of 0. Circulation rates increase with d. Accordingly, problem 2 gives the lowest circulation rates. Flow rates in problems 3 and 4 are very similar, and they are lower than in case 1. Also, the circulation rates in problem 4 exceed those in problem 1 only at $Ra = 10^3$, where conduction is dominant. Circulation rates in case 5 exceed those in problem 2 only.

As the vertical walls are partially active, the flow departs from them in the inactive zones. In all cases, therefore, the flow is more elliptic than in the reference case. Over the whole range of Ra, the circulation is weaker than that for the test problem, except for case 1 at Rayleigh numbers from approximately 10^4. The existence of thicker boundary layers in part of the perimeter results in smaller velocity gradients and lower frictional forces. Also, the flow leaving an active region is free to move vertically, while in the test problem the restrictions imposed by the horizontal walls act on the fluid in the boundary layers. These facts account for the higher circulation rates found in problem 1 (although the active area is less than that for the test problem) at Rayleigh numbers above 10^4.

4.2. *Temperature fields and heat transfer*

It is useful to consider the conduction regime first. In pure conduction, where heat transfer in situations 1 and 2 is the same, $Nu_0 = 1.345$. In problems 3, 4 and 5, Nu_0 values are 1.400, 1.620 and 1.487, respectively. The heat transfer in case 4 is the highest because this situation gives the minimum distance between points on the hot and cold surfaces. Also, in all cases, Nu_0 exceeds the value for the test problem in pure conduction. This is because in the five problems, the available heat transfer area at the vertical symmetry plane is twice the active area of each side, while in the test problems these areas are equal.

At $Ra = 10^3$, isotherms are nearly vertical in the vicinity of the vertical midplane, except in cases 2 and 5 where they are inclined. Nu_0 in case 4 is the highest, followed by the Nusselt numbers in cases 5, 3 and 1, in the same order as in pure conduction. All five situations give higher heat transfer rates than the test problem.

At $Ra = 10^6$, the temperature becomes stratified in different ways according to each case. In case 1 (Fig. 3(a)), the isotherms show an inverse slope, giving rise to higher wall temperature gradients than in the other situations. In case 2 (Fig. 3(b)), stratification is almost complete in a central strip, with hot and cold regions above and below the stratified zone. In case 3 (Fig. 3(c)), a nearly isothermal hot region above $Y = 0.5$ develops. Below this line, thermal stratification occurs. In problem 4 (Fig. 3(d)), the temperature field is close to case 2, but with bigger temperature gradients at the active walls. The behaviour of isotherms in case 5 (Fig. 3(e)) is intermediate between cases 2 and 4.

The average heat transfer is summarized in Fig. 4. In spite of the different circulation rates, the heat transfer does not vary significantly from situations 3 and 4 to situation 1. Curves for all these cases tend to merge at $Ra = 10^7$. Problem 3 has consistently lower heat transfer rates than problem 1. In problem 5, in which Nusselt numbers are very high at $Ra = 10^3$, the heat transfer is lower than in all cases (except case 2) at high Rayleigh numbers.

Cases with low circulation show a lower slope in the Nu_0–Ra curve (e.g. cases 2–4). As observed in Section 4.1, the boundary layer regime is not completely developed even at high Ra. As a consequence, the heat transfer still shows partially the characteristics of the conduction regime even at very high Rayleigh numbers. This explains why, in situations like 4 and 1, with very different circulation rates, the overall Nusselt numbers at $Ra = 10^7$ are not so different.

The heat transfer results for $Ra = 10^3$–10^7 can be described by a power law of the type

$$Nu_0 = C\,Ra^b. \tag{6}$$

Values of C and b are given in Table 2. The maximum deviations from the predictions of equation (6) are $\pm 0.6\%$ in problem 4, $\pm 2.1\%$ in problem 1 and $\pm 6\%$ in the other problems.

A further generalization of heat transfer results can be made. Consider problem 1 (Fig. 1(a)). If the cold region is moved downwards, problem 3 is obtained. At each Ra, the value of Nu_0 at an intermediate position will be between the values of this parameter for problems 1 and 3. Similarly, the heat transfer in intermediate cases between 1 and 4 and between 3 and 4 can be estimated. In addition, by moving both active regions of problem 3 upwards, a new case (called 3′, with the active regions at the top of their respective walls) can be defined. Cases 3 and 3′ are exactly equivalent. Thus the heat transfer in every case where the total exit length is between 0 and 1 can be estimated by equation (6). Cases 4 and 2 represent the upper and lower limits of heat transfer over most of the range of Rayleigh numbers considered.

A general heat transfer correlation would contain d as an additional parameter. However, a single expression for Nu_0 is hard to obtain because increases in d do not always produce increases in Nu_0. This is because, although heat transfer rates tend to become equal at very high Ra for the cases in which $0.5 < d < 1$, the effect of the different heat transfer levels for conduction is felt at intermediate Rayleigh numbers. Conversely, flow rates, measured by the maximum value of ψ, increase definitely with increases in d.

It is useful to compare the results of the present work with those of Poulikakos [3] for square cavities. Except for the fact that in ref. [3] heaters and coolers

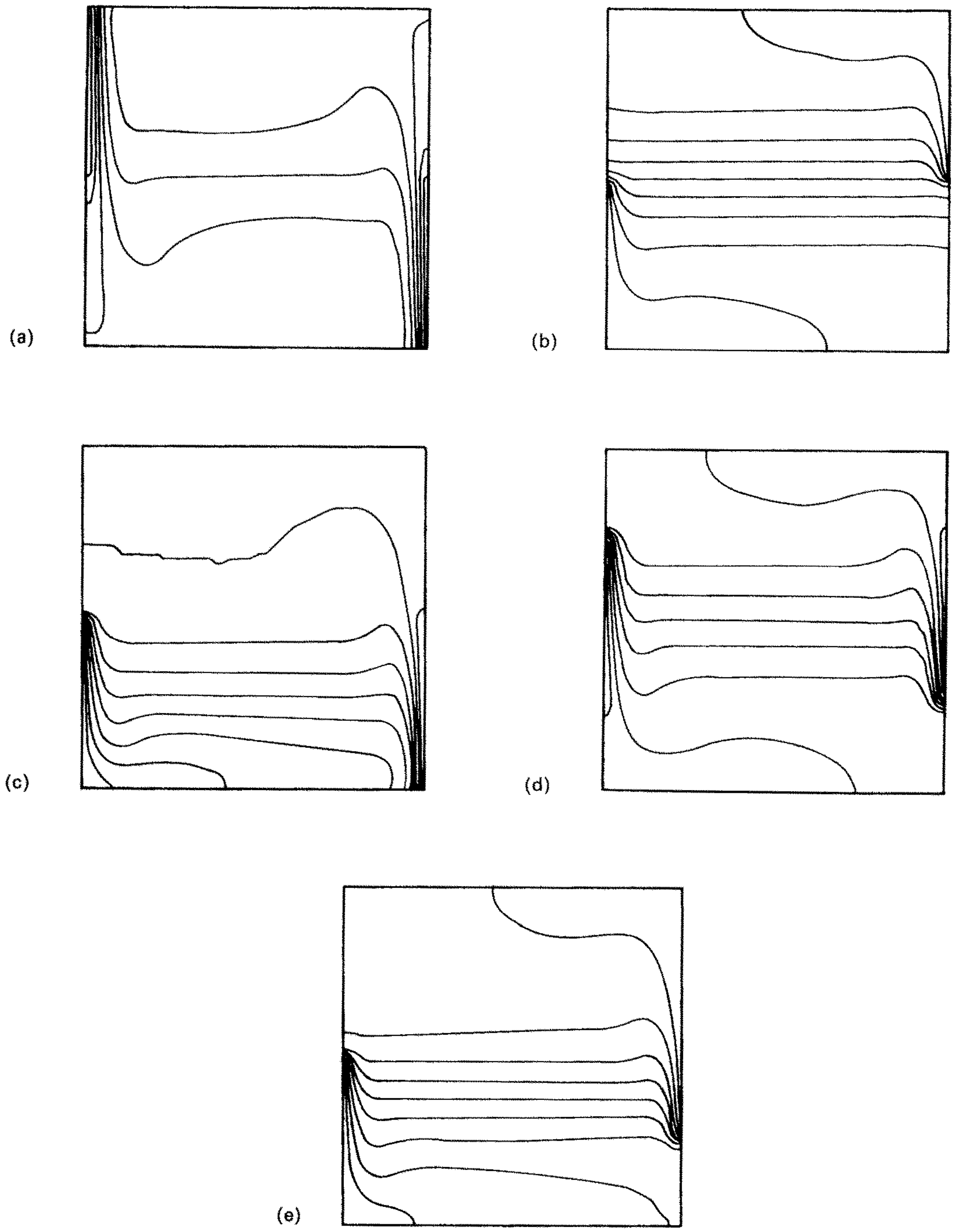

FIG. 3. Isotherms, $Ra = 10^6$: (a) case 1; (b) case 2; (c) case 3; (d) case 4; (e) case 5. Equally spaced isotherms, from $\Theta = 0$ to 1.

are on the same wall, the case termed 'side heating below side cooling' is analogous to case 1. Nusselt numbers from ref. [3] exceed ours at low Ra, while at $Ra = 10^4$–10^5 the reverse is true. Besides, 'side heating above side cooling' is analogous to case 2. Here again, average Nusselt numbers from ref. [3] are lower than ours at Rayleigh numbers of 10^4–10^5. Differences in flow pattern explain this behaviour. In ref. [3], a two-cell flow is always observed, without exchange of fluid between the upper and lower cavity halves. In our problems the fluid always crosses the horizontal mid-plane. This mixing, albeit small in situation 2, increases the heat transfer relative to the corresponding cases in ref. [3]. Poulikakos does not give results for $Ra > 10^5$, although he suggests that instabilities would occur at higher Ra. These instabilities will

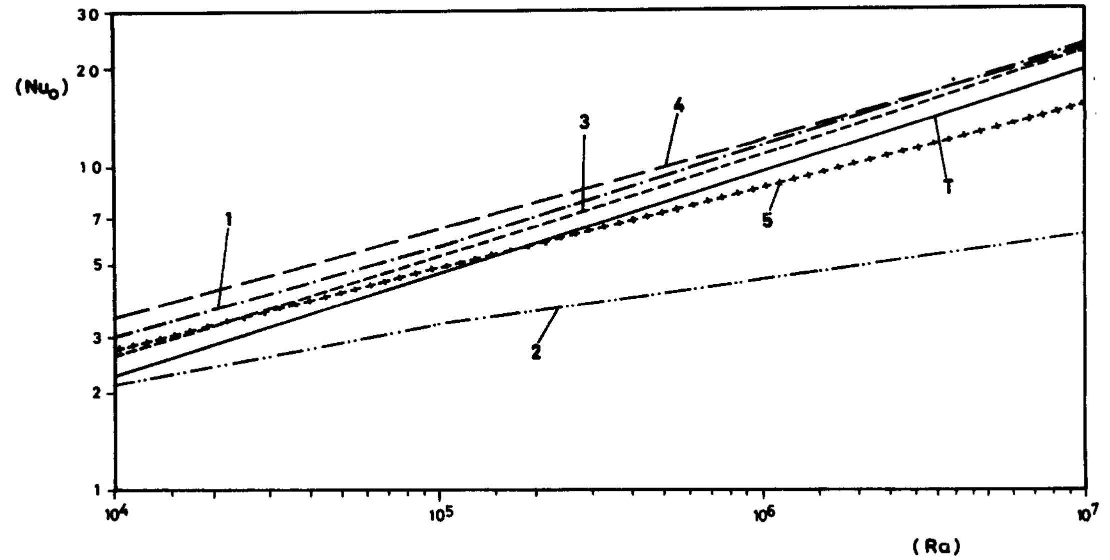

FIG. 4. Average Nusselt number (Nu_0) vs Rayleigh number for cases 1–5 and test case.

Table 2. Constants C and b in equation (6)

Case	Test	1	2	3	4	5
C	0.132	0.185	0.495	0.184	0.266	0.295
b	0.310	0.300	0.159	0.296	0.276	0.244

probably result in some fluid crossing the horizontal midplane when side heating is located below side cooling.

The results of this work show that the heat transfer rates, in the applications which motivate this study, can be controlled to a certain extent by varying the relative positions of the hot and cold elements. Further study is needed in order to establish limits for heat transfer in rectangular cavities with active elements of arbitrary lengths and relative positions.

5. CONCLUSIONS

Five cases of natural convection in a square cavity with half-heated and half-insulated vertical walls were studied numerically. Although conduction and boundary layer regimes occur, at high Rayleigh numbers the flow near the unheated vertical walls loses part of its boundary layer characteristics, owing to the absence of buoyancy forces there. The circulation rates vary strongly with the total available length downstream of the active regions.

Average Nusselt numbers are less dependent than flow rates on the total exit length for $0.5 < d < 1$. Differences in heat transfer, caused by conduction, are still noticed among the different cases even at Rayleigh numbers as high as 10^5. Nusselt numbers are higher than in the test problem at all values of Ra in cases 1, 3 and 4. In the other cases, owing to the more restrictive nature of the boundary conditions, Nusselt numbers are lower than those for the test problem at high Rayleigh numbers.

Expressions for average Nusselt number in the five cases, valid to within $\pm 6\%$ in the worst case, are given. Problems 4 and 2 represent the upper and lower limits of heat transfer. The average heat transfer in a square cavity with half-active vertical walls and with the active sectors in any relative positions in which $d \geqslant 0.5$ can be estimated from the results obtained for the five cases analysed. Some suggestions for further work are advanced.

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TRANSFERT THERMIQUE DANS DES CAVITES CARREES AVEC DES PAROIS VERTICALES PARTIELLEMENT ACTIVES

Résumé—On étudie numériquement la convection naturelle de l'air dans des cavités carrées avec des parois verticales à moitié adiabatiques, pour des nombres de Rayleigh entre 10^3 et 10^7. Ce problème est relié aux applications solaires et au refroidissement des composants électroniques. Cinq positions relatives différentes des zones actives sont considérées. Alors que la circulation dépend fortement de la longueur totale de sortie des zones actives, le transfert de chaleur dépend moins de ce paramètre. Des effets significatifs de conduction apparaissement au nombre de Rayleigh égal à 10^5. On donne des expressions du nombre de Nusselt moyen pour les cinq situations et la possibilité de les généraliser aux cas intermédiaires.

WÄRMEÜBERGANG IN QUADRATISCHEN HOHLRÄUMEN MIT TEILWEISE BEHEIZTEN SENKRECHTEN WÄNDEN

Zusammenfassung—Die Naturkonvektion von Luft in quadratischen Hohlräumen, deren senkrechte Wände teilweise beheizt, teilweise adiabat sind, wird für Rayleigh-Zahlen von 10^3 bis 10^7 numerisch untersucht. Diese Fragestellung steht im Zusammenhang mit der Solarenergienutzung und der Kühlung elektronischer Bauteile. Fünf verschiedene Positionen der beheizten Zonen werden betrachtet. Während die Zirkulation stark von der gesamten Abströmlänge hinter den beheizten Zonen abhängt, trifft dies für den Wärmeübergang weniger zu. Die Wärmeleitung ist sogar bei einer Rayleigh-Zahl von 10^5 noch bedeutsam. Es werden Ausdrücke für die mittlere Nusselt-Zahl bei den fünf verschiedenen Positionen angegeben, außerdem Hinweise, wie die Ergebnisse für den Wärmeübergang für dazwischenliegende Fälle übertragen werden können.

ТЕПЛОПЕРЕНОС В КВАДРАТНЫХ ПОЛОСТЯХ С ЧАСТИЧНО АКТИВНЫМИ ВЕРТИКАЛЬНЫМИ СТЕНКАМИ

Аннотация—Численно исследуется естественная конвекция воздуха в квадратных полостях с наполовину активными и наполовину изолированными вертикальными стенками при значениях числа Рэлея 10^3–10^7. Такая задача возникает при исследовании аккумуляторов солнечной энергии и охлаждения элементов электронных устройств. Рассмотрено пять различных относительных расположений активных зон. В то время как конвекция сильно зависит от полной длины активной зоны вниз по течению, теплоперенос меньше зависит от этого параметра. Влияние теплопроводности наблюдается даже при числе Рэлея, равном 10^5. Приведены выражения для среднего числа Нуссельта в пяти указанных случаях, даны рекомендации по обобщению результатов по теплообмену для промежуточных ситуаций.